New Concepts in Team Theory:
Mean Field Teams & Reinforcement Learning

Jalal Arabneydi

PhD Oral Defense: December 8, 2016

Electrical and Computer Engineering department, McGill University

Introduction to Team Theory

Team theory studies decision makers that wish collaborate to accomplish a common task.

Team theo… laborate to

accomplish…

Salient feature of Teams:

- Multiple decision makers.
- Decentralized information.
- Common objective.

Team in various applications

- Networked control
- Robotics
- Communication
- Transportation
- Sensor networks
- Smart grids
- Economics
- etc.

Team in various applications

- Networked control
- Robotics
- Communicatio
- Transportation
- Sensor networ
- Smart grids
- Economics
- etc.

Teams are almost everywhere.

Background of team theory

Static team (Radner 1962, Marschack and Radner 1972)

Dynamic team (Witsenhausen 1971, Witsenhausen 1973)

Specific information structure

- Partially nested (Ho and Chu 1972)
- One-step delayed sharing (Witsenhausen 1971, Yoshikawa 1978)
- n-step delayed sharing (Witsenhausen 1971, Varaiya 1978, Nayyar 2011)
- Common past sharing (Aicardi 1978)
- Periodic sharing (Ooi 1997)
- Belief sharing (Yuksel 2009)
- Partial history sharing (Nayyar 2013)

Motivation

- Explicit optimal solutions typically for 2-3 agents:
  big gap between theory and application.
- When the model is not known completely:
  no optimal result even for 2-3 agents.

Motivation

- Explicit optimal solutions typically for 2-3 agents: **big gap between theory and application.**
- **Mean Field Teams.**
- When the model is not known completely: **no optimal result even for 2-3 agents.**
- **Reinforcement Learning w.t. partial history sharing.**

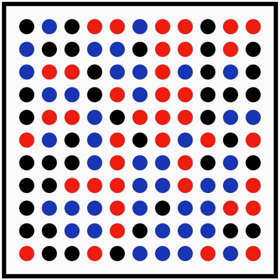
Mean Field Teams

Partially exchangeable agents

Smart grids

Swarm robotics

Social networks



- $\mathcal{N}$: set of heterogeneous agents
- $\mathcal{K}$: set of sub-populations

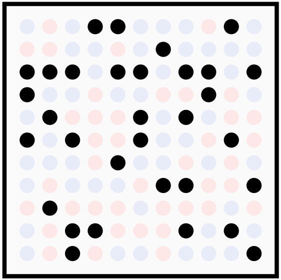
For entire population:

- $\mathbf{x}_t$: joint state at time t
- $\mathbf{u}_t$: joint action at time t

For agent i of sub-population $k \in \mathcal{K}$:

- $\mathcal{N}^k$: entire sub-population of type $k \in \mathcal{K}$
- $x_t^i \in \mathcal{X}^k$: state of agent i at time t
- $u_t^i \in \mathcal{U}^k$: action of agent i at time t

Notation



- $\mathcal{N}$: set of heterogeneous agents
- $\mathcal{K}$: set of sub-populations

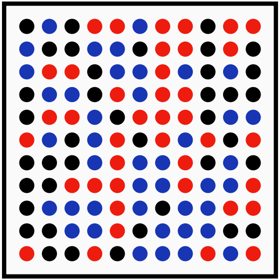
For entire population:

- $\mathbf{x}_t$: joint state at time t
- $\mathbf{u}_t$: joint action at time t

For agent *i* of sub-population $k \in \mathcal{K}$:

- $\mathcal{N}^k$: entire sub-population of type $k \in \mathcal{K}$
- $x_t^i \in \mathcal{X}^k$: state of agent *i* at time *t*
- $u_t^i \in \mathcal{U}^k$: action of agent *i* at time *t*

Partially exchangeable agents



Definition (Exchangeable agents)

A pair (i, j) of agents is exchangeable if:

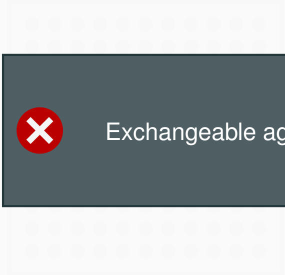
1) For any t, and any $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{w}$,

$$\sigma_{i,j}\big(f_t(\mathbf{x}, \mathbf{u}, \mathbf{w})\big) = f_t(\sigma_{i,j}\mathbf{x}, \sigma_{i,j}\mathbf{u}, \sigma_{i,j}\mathbf{w}),$$

2) For any t, and any $\mathbf{x}$ and $\mathbf{u}$,

$$c_t(\mathbf{x}, \mathbf{u}) = c_t(\sigma_{i,j}\mathbf{x}, \sigma_{i,j}\mathbf{u}),$$

Partially exchangeable agents



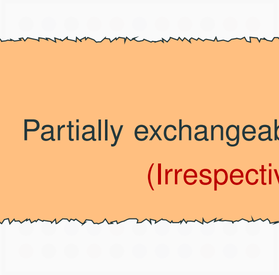
Definition (Exchangeable agents)

A pair (i, j) of agents is exchangeable if:

$$c_t(\mathbf{x}, \mathbf{u}) = c_t(\sigma_{i,j}\mathbf{x}, \sigma_{i,j}\mathbf{u}),$$

Exchangeable agents $\not\Longleftrightarrow$ Exchangeable initial states & noises

Partially exchangeable agents



Definition (Exchangeable agents)

A pair (i, j) of agents is exchangeable if:

$$c_t(\mathbf{x}, \mathbf{u}) = c_t(\sigma_{i,j}\mathbf{x}, \sigma_{i,j}\mathbf{u}),$$

Partially exchangeable agents $\equiv$ Mean-field coupled agents

(Irrespective of information structure)

Mean field models: controlled Markov chain

Suppose the dynamics $\mathbf{x}_{t+1} = f_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{w}_t)$.

The per-step cost is $c_t(\mathbf{x}_t, \mathbf{u}_t)$.

Proposition 2.2

There exist functions $\{\{f_t^k\}_{k\in\mathcal{K}}, \ell_t\}$ such that for agent $i \in \mathcal{N}^k$

$$\boxed{x_{t+1}^i = f_t^k(x_t^i, u_t^i, \boldsymbol{\xi}_t, w_t^i),}$$

and the per-step cost at time t, may be written as

$$\boxed{\ell_t(\boldsymbol{\xi}_t).}$$

$$\mathbf{m}_t = \operatorname{vec}(m_t^1, \dots, m_t^K), \qquad \boldsymbol{\xi}_t = \operatorname{vec}(\xi_t^1, \dots, \xi_t^K),$$

$$m_t^k = \frac{1}{|\mathcal{N}^k|} \sum_{i\in\mathcal{N}^k} \delta_{x_t^i}, \qquad \xi_t^k = \frac{1}{|\mathcal{N}^k|} \sum_{i\in\mathcal{N}^k} \delta_{x_t^i, u_t^i}.$$

Mean-field models: linear quadratic

Suppose the dynamics are linear, i.e., $\mathbf{x}_{t+1} = A_t \mathbf{x}_t + B_t \mathbf{u}_t + \mathbf{w}_t$.

The per-step cost is quadratic, i.e., $c_t(\mathbf{x}_t, \mathbf{u}_t) = \mathbf{x}_t^\intercal Q_t \mathbf{x}_t + \mathbf{u}_t^\intercal R_t \mathbf{u}_t$.

Proposition 2.1

There exist matrices $\{A_t^k, B_t^k, D_t^k, E_t^k, Q_t^k, R_t^k\}_{k \in \mathcal{K}}$ and P_t^x and P_t^u such that

$$\boxed{x_{t+1}^i = A_t^k x_t^i + B_t^k u_t^i + D_t^k \bar{\mathbf{x}}_t + E_t^k \bar{\mathbf{u}}_t + w_t^i.}$$

and the per-step cost at time t, may be written as

$$\boxed{\bar{\mathbf{x}}_t^\intercal P_t^x \bar{\mathbf{x}}_t + \bar{\mathbf{u}}_t^\intercal P_t^u \bar{\mathbf{u}}_t + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}^k} \frac{1}{|\mathcal{N}^k|} \Big[(x_t^i)^\intercal Q_t^k x_t^i + (u_t^i)^\intercal R_t^k u_t^i \Big].}$$

$$\bar{\mathbf{x}}_t = \operatorname{vec}(\bar{x}_t^1, \dots, \bar{x}_t^K), \qquad \bar{\mathbf{u}}_t = \operatorname{vec}(\bar{u}_t^1, \dots, \bar{u}_t^K),$$

$$\bar{x}_t^k = \frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} x_t^i, \qquad \bar{u}_t^k = \frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} u_t^i.$$

Mean-field teams: problem formulation

Controlled Markov Chain

- Dynamics: $x^i_{t+1} = f^k_t(x^i_t, u^i_t, \boldsymbol{\xi}_t, w^i_t)$
- Per-step cost: $\ell_t(\boldsymbol{\xi}_t)$
- Information structure: $u^i_t = g^i_t(x^i_t, \mathbf{m}_{1:t})$
- Objective:
  $J^* = min_{\mathbf{g}} \left(\mathbb{E}^{\mathbf{g}} \left[\sum_{t=1}^{T} \ell_t(\boldsymbol{\xi}_t) \right] \right)$

Linear Quadratic

- $x^i_{t+1} = A^k_t x^i_t + B^k_t u^i_t + D^k_t \bar{\mathbf{x}}_t + E^k_t \bar{\mathbf{u}}_t + w_t$
- $\ell_t(\mathbf{x}_t, \mathbf{u}_t) = \bar{\mathbf{x}}_t^{\intercal} P^x_t \bar{\mathbf{x}}_t + \bar{\mathbf{u}}_t^{\intercal} P^u_t \bar{\mathbf{u}}_t +$
  $\sum_{k=1}^{K} \sum_{i \in \mathcal{N}^k} \frac{1}{|\mathcal{N}^k|} \left[(x^i_t)^{\intercal} Q^k_t x^i_t + (u^i_t)^{\intercal} R^k_t u^i_t \right]$
- $u^i_t = g^i_t(x^i_t, \bar{\mathbf{x}}_{1:t})$
- $J^* = \inf_{\mathbf{g}} \left(\mathbb{E}^{\mathbf{g}} \left[\sum_{t=1}^{T} \ell_t(\mathbf{x}_t, \mathbf{u}_t) \right] \right).$

Mean-field teams: key assumptions

Controlled Markov Chain

A 4.1 The control laws are exchangeable i.e. $g_t^i = g_t^j$ for any $i, j \in \mathcal{N}^k$.

It is a standard assumption in large scale systems for reasons: simplicity, fairness, & robustness.

Linear Quadratic

Not needed.

Mean-field teams: key assumptions

Controlled Markov Chain

Linear Quadratic

A 1.1. The control laws are exchange

No assumptions on the probability distributions across agents.

- Gaussian or non-Gaussian,
- Independent or highly correlated,
- Exchangeable or non-exchangeable.

large scale systems for reasons: simplicity, fairness, & robustness.

Mean-field teams: main challenges

Controlled Markov Chain

- Coupling in dynamic and cost with non-classical information structure. This belongs to NEXP.

- Designer's approach, impractical dynamic program.

- Common information approach, state space of dynamic program increases exponentially in number of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

Linear Quadratic

Mean-field teams: main challenges

Controlled Markov Chain

- Coupling in dynamic and cost with non-classical information structure. This belongs to NEXP.
- Designer's approach, impractical dynamic program.
- Common information approach, state space of dynamic program increases exponentially in number of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

Linear Quadratic

- LQG with non-classical information structure is difficult.
- Linear strategies are optimal only for Gaussian and partially nested.
- The mean field sharing is not partially nested and the noises are allowed to be non-Gaussian.

Mean-field teams: main challenges

Controlled Markov Chain

Linear Quadratic

Witsenhausen's counterexample is still an open problem after 48 years!

- Designer's approach, impractical dynamic program.
- Common information approach, state space of dynamic program increases exponentially in number of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

- Linear strategies are optimal only for Gaussian and partially nested.
- The mean field sharing is not partially nested and the noises are allowed to be non-Gaussian.

Mean-field teams: main challenges

Controlled Markov Chain

Linear Quadratic

Witsenhausen's counterexample is still an open problem after 48 years!

Lipsa and matrins, Optimal memoryless control in Gaussian noise: A simple counterexample, 2008.

- Common information approach, state space of dynamic program increases exponentially in number of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

- The mean field sharing is not partially nested and the noises are allowed to be non-Gaussian.

Mean-field teams: main challenges

Controlled Markov Chain

Linear Quadratic

Witsenhausen's counterexample is still an open problem after 48 years!

Lipsa and matrins, Optimal memoryless control in Gaussian noise: A simple counterexample, 2008.

Yuksel and Tatikonda, A counterexample in distributed optimal sensing, 2009.

ber of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

Mean-field teams: main challenges

Controlled Markov Chain | Linear Quadratic

- Witsenhausen's counterexample is still an open problem after 48 years!
- Lipsa and matrins, Optimal memoryless control in Gaussian noise: A simple counterexample, 2008.
- Yuksel and Tatikonda, A counterexample in distributed optimal sensing, 2009.
- Whittle and Rudge, The optimal linear solution of a symmetric team control problem, 1974.

Mean-field teams: main challenges

Controlled Markov Chain

- Coupling in dynamic and cost with non-classical information structure. This belongs to NEXP.
- Designer's approach, impractical dynamic program.
- Common information approach, state space of dynamic program increases exponentially in number of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

Linear Quadratic

- LQG with non-classical information structure is difficult.
- Linear strategies are optimal only for Gaussian and partially nested.
- The mean field sharing is not partially nested and the noises are allowed to be non-Gaussian.

Mean-field teams: main challenges

Controlled Markov Chain

- Coupling in dynamic and cost with non-classical information structure. This belongs to NEXP.
- Common information approach; state space of dynamic program increases exponentially in number of agents and time, i.e., $\mathbb{P}(x_t^1, \ldots, x_t^N \mid \mathbf{m}_{1:t})$.

Linear Quadratic

- LQG with non-classical information structure is difficult.
- The mean field sharing is not partially nested and the noises are allowed to be non-Gaussian.

There is no existing approach to solve mean-field teams.

Mean-field teams: main theorems

Theorem 4.1

Define recursively value functions:

$$V_{T+1}(\mathbf{m}) = 0, \quad \mathbf{m} \in \mathcal{M}_n,$$

and for $t = T, \dots, 1$, for $\mathbf{m} \in \mathcal{M}_n$,

$$\boxed{V_t(\mathbf{m}) = \min_{\boldsymbol{\gamma}} \mathbb{E}\left[\ell_t\left(\phi(\mathbf{m}_t, \gamma_t)\right) + V_{t+1}(\mathbf{m}_{t+1}) \mid \mathbf{m}_t = \mathbf{m}, \gamma_t = \boldsymbol{\gamma}\right],}$$

where $\boldsymbol{\gamma} = (\gamma^1, \dots, \gamma^K), \gamma^k : \mathcal{X}^k \to \mathcal{U}^k$, and

$$\phi(\mathbf{m}, \gamma)(\mathbf{x}, \mathbf{u}) = \mathbf{m}(\mathbf{x}) \prod_{k=1}^{K} \mathbb{1}\left(u^k = \gamma^k(x^k)\right), \mathbf{x} \in \prod_{k=1}^{K} \mathcal{X}^k, \mathbf{u} \in \prod_{k=1}^{K} \mathcal{U}^k, x^k \in \mathcal{X}^k, u^k \in \mathcal{U}^k$$

Let ψ_t^* denote any argmin of the right hand side. Then, optimal solution is

$$g_t^{*,k}(\mathbf{m}, x) := \psi_t^{*,k}(\mathbf{m})(x), \quad \mathbf{m} \in \mathcal{M}_n, x \in \mathcal{X}^k, k \in \mathcal{K}.$$

Mean-field teams: main theorems

Theorem 3.1

The optimal strategy is unique, identical across sub-populations, and is linear in local state and the mean-field of the system. In particular,

$$\boxed{u_t^i = \breve{L}_t^k (x_t^i - \bar{x}_t^k) + \bar{L}_t^k \bar{\mathbf{x}}_t, \quad i \in \mathcal{N}^k, k \in \mathcal{K},}$$

where the above gains are obtained by the solution of $K+1$ Riccati equations: one for computing each $\breve{L}_t^k$, $k \in \mathcal{K}$, and one for $\bar{L}_t := \operatorname{vec}(\bar{L}_t^1, \dots, \bar{L}_t^K)$. Let $\breve{M}_{1:T}^k$ and $\bar{M}_{1:T}$ denote the solution of the above Riccati equations and

$$\breve{\Sigma}_t^k := \frac{\sum_{i \in \mathcal{N}^k} \operatorname{var}(w_t^i - \bar{w}_t^k)}{|\mathcal{N}^k|}, \bar{\Sigma}_t := \operatorname{var}(\bar{\mathbf{w}}_t), \breve{\Xi}^k := \frac{\sum_{i \in \mathcal{N}^k} \operatorname{var}(x_1^i - \bar{x}_1^k)}{|\mathcal{N}^k|}, \bar{\Xi} := \operatorname{var}(\bar{\mathbf{x}}_1)$$

Then, the optimal cost is given by

$$J^* = \sum_{k \in \mathcal{K}} \operatorname{Tr}(\breve{\Xi}^k \breve{M}_1^k) + \operatorname{Tr}(\bar{\Xi} \bar{M}_1) + \sum_{t=1}^{T-1} \left[\sum_{k \in \mathcal{K}} \operatorname{Tr}(\breve{\Sigma}_t^k \breve{M}_{t+1}^k) + \operatorname{Tr}(\bar{\Sigma}_t \bar{M}_{t+1}) \right].$$

Mean-field teams: main theorems

Theorem 3.1

The optimal strategy is unique, identical across sub-populations, and is linear in local state and the mean-field of the system. In particular,

$$u_t^i = \breve{L}_t^k (x_t^i - \bar{x}_t^k) + \bar{L}_t^k \bar{\mathbf{x}}_t, \quad i \in \mathcal{N}^k, k \in \mathcal{K},$$

For agent $i \in \mathcal{N}^k$ in sub-population $k \in \mathcal{K} = \{1, \ldots, K\}$,

$$u_t^i = g_t^{*,k}(\mathbf{m}_t, x_t^i), \qquad u_t^i = \breve{L}_t^k (x_t^i - \bar{x}_t^k) + \bar{L}_t^k \bar{\mathbf{x}}_t.$$

Then, the optimal cost is given by

$$J^* = \sum_{k \in \mathcal{K}} \operatorname{Tr}(\breve{\Xi}^k \breve{M}_1^k) + \operatorname{Tr}(\bar{\Xi} \bar{M}_1) + \sum_{t=1}^{T-1} \Big[\sum_{k \in \mathcal{K}} \operatorname{Tr}(\breve{\Sigma}_t^k \breve{M}_{t+1}^k) + \operatorname{Tr}(\bar{\Sigma}_t \bar{M}_{t+1}) \Big].$$

Salient features

Controlled Markov Chain

- The solution complexity is polynomial in number of agents (rather than exponential) and linear in time (rather than exponential.)

Linear Quadratic

- No need to share anything beyond mean field.
- The solution complexity depends on the number of sub-populations, i.e., K but **not on the number of agents in each sub-population, i.e., $\mathcal{N}^k$**.
- Each agent needs to solve only two Riccati equations (distributed computation).

Mean-field teams: generalizations

Controlled Markov Chain

- **Arbitrarily coupled cost**
- **Infinite horizon**
- **Noisy observation**
- **Major-minor**
- **Randomized strategies**

Linear Quadratic

- **Weighted mean field**
- **Infinite horizon**
- **Partial mean field sharing**
- **Major-minor**
- **Tracking problem**

Controlled Markov Chain

Linear Quadratic

Within the same sub-population, each agent is allowed to have different tracking reference and weights:

$$u_t^i = \breve{L}_t^k(x_t^i - \lambda^i \bar{x}_t^{k,\lambda}) + \lambda^i \bar{L}_t^k \bar{\mathbf{x}}_t^\lambda + \breve{F}_t^k v_t^{i,\lambda^i} + \lambda^i \bar{F}_t^k \bar{v}_t^\lambda$$

- Randomized strategies

- Tracking problem

Mean-field teams: generalizations

Controlled Markov Chain

- **Arbitrarily coupled cost**
- **Infinite horizon**
- **Noisy observation**
- **Major-minor**
- **Randomized strategies**

Linear Quadratic

- **Weighted mean field**
- **Infinite horizon**
- **Partial mean field sharing**
- **Major-minor**
- **Tracking problem**

Mean-field teams: generalizations

When mean field of only sub-populations $\mathcal{S} \in \mathcal{K}$ are observed:

$$u_t^i = \breve{L}_t^k (x_t^i - z_t^k) + \bar{L}_t^k \mathbf{z}_t,$$

where
$$z_{t+1}^k = \begin{cases} \bar{x}_{t+1}^k, & k \in \mathcal{S}, \\ A_t^k z_t^k + (B_t^k \bar{L}_t^k + D_t^k + E_t^k \bar{L}_t)\mathbf{z}_t, & k \in \mathcal{S}^c. \end{cases}$$

The approximation error

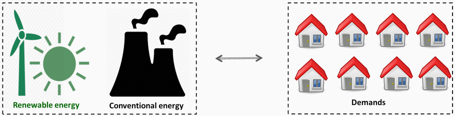
$$\Delta J = \operatorname{Tr}(\tilde{X}_1 \tilde{M}_1) + \sum_{t=1}^{T-1} \operatorname{Tr}(\tilde{W}_t \tilde{M}_{t+1}),$$

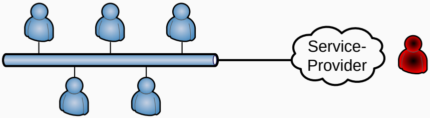
where $\tilde{M}_{1:T}$ is the solution of a Lyapunov equation. It is bounded as

$$\boxed{\Delta J \in \mathcal{O}\left(\frac{T}{n}\right).}$$

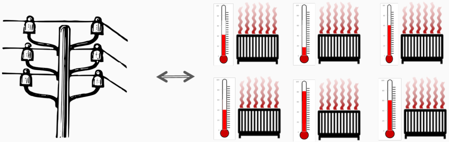
Mean-field teams: numerical examples

Controlled Markov Chain

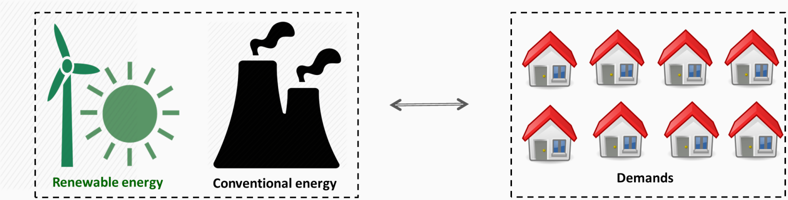




Linear Quadratic



Numerical example 1: demand response



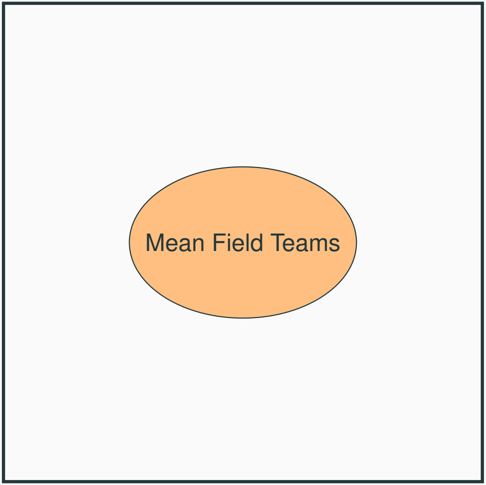
- $x_t^i \in \mathcal{X} = \{OFF, ON\}, \quad m_t = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}(x_t^i = OFF)$
- Dynamics: $\mathbb{P}(x_{t+1}^i | x_t^i, u_t^i) =: [P(u_t^i)]_{x_t^i x_{t+1}^i}$
- Actions: $u_t^i \in \mathcal{U} = \{FREE, OFF, ON\}$, Cost of action: $C(u_t^i)$
- Objective: $\min_{\mathbf{g}} \mathbb{E}^{\mathbf{g}} \left[\sum_{t=1}^{\infty} \beta^t \left(\frac{1}{n}\sum_{i=1}^{n} C(u_t^i) + D(m_t \| \zeta_t)\right)\right]$.

Numerical example 1: demand response

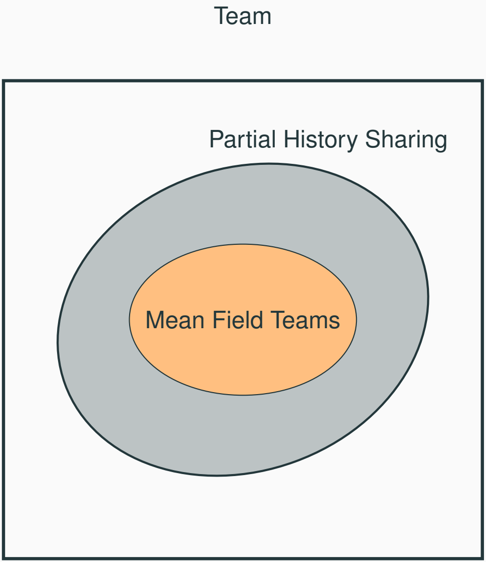
Reinforcement Learning with Partial History Sharing

Reinforcement learning with partial history sharing

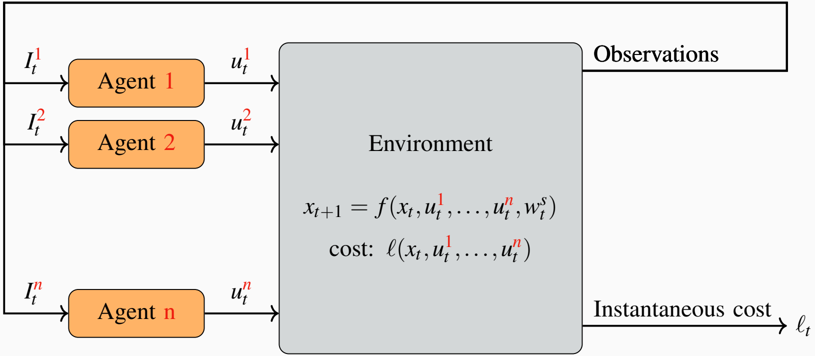
Team



Reinforcement learning with partial history sharing



Reinforcement learning in general team



State: $x_t \in \mathcal{X}$. Observation: $y_t^i = h(x_t, u_{t-1}^1, \dots, u_{t-1}^n, w_t^{i,o})$.

Control law: $u_t^i = g_t^i(I_t^i)$. Information: $I_t^i \subseteq \{y_{1:t}^1, \dots, y_{1:t}^n, u_{1:t-1}^1, \dots, u_{1:t-1}^n\}$.

System cost: Given $\beta \in (0,1)$, $J(\boldsymbol{g}) = \mathbb{E}^{\boldsymbol{g}}\left[\sum_{t=1}^{\infty} \beta^{t-1} \ell(x_t, u_t^1, \dots, u_t^n)\right]$.

Suppose the system dynamics (f, h), cost structure ℓ, and probability mass functions are not completely known.

Objective: Given $\epsilon > 0$, find strategy $\boldsymbol{g}^*_\epsilon$ such that

$$J(\boldsymbol{g}^*_\epsilon) \leq J^* + \epsilon.$$

Reinforcement learning with Partial History Sharing (PHS)

Definition (Partial History Sharing, Nayyer et al. 2013)

Split the information at each agent into two parts:

- *Common information:* $c_t = \bigcap_{i=1}^{n} I_t^i$ i.e. shared between all agents.
- *Local information:* $m_t^i = I_t^i \backslash c_t$ that is the local information of agent i.

Define $z_t := c_{t+1} \backslash c_t$ as common observation, hence $c_{t+1} = z_{1:t}$. Then,

a) The update of local information

$$\boxed{m_{t+1}^i \subseteq \{m_t^i, u_t^i, y_{t+1}^i\} \backslash z_t, \quad i \in \{1, \ldots, n\}.}$$

b) For every agent i, $|m_t^i|$ and $|z_t|$ are uniformly bounded in time t.

PHS encompasses: **delayed sharing**, **mean-field sharing**, **periodic sharing**, **control sharing**, etc.

Reinforcement learning in team: main challenges

Given centralized MDP, there are two ways to learn the optimal solution:

- **Indirect**: supervised learning and dynamic program.
- **Direct (Reinforcement Learning)**: Barto, Sutton, Watkins, Dayan, Singh, etc. (active since 80's).

Reinforcement learning in team: main challenges

Most of existing RL methods are developed for finite state-action MDPs. However, decentralized systems are **not** MDP in general.

- **Indirect**: supervised learning and dynamic program.
- **Direct (Reinforcement Learning)**: Barto, Sutton, Watkins, Dayan, Singh, etc. (active since 80's).

Reinforcement learning in team: main challenges

Most of existing RL methods are developed for finite state-action MDPs. However, decentralized systems are **not** MDP in general.

The **indirect method may not be feasible** due to the incomplete information i.e. dynamics and cost may not be fully identified.

etc. (active since 80's).

Reinforcement learning in team: main challenges

Most of existing RL methods are developed for finite state-action MDPs. However, decentralized systems are **not** MDP in general.

The **indirect method may not be feasible** due to the incomplete information i.e. dynamics and cost may not be fully identified.

[Nayyer et al. 2013] identifies a dynamic program for PHS; however,

- The state space is an infinite set.
- The state space depends on the model.

There is no RL algorithm for POMDP that guarantees optimality.

Reinforcement learning in team: main challenges

Given centralized MDP, there are two ways to learn the optimal solution:

No existing approach to solve decentralized reinforcement learning.

etc. (active since 80's).

Pre-learning stage

STEP 1: Common Information Approach

Define partial function $\gamma_t^i : \mathcal{M}^i \to \mathcal{U}^i$:

$$\gamma_t^i := g_t^i(z_{1:t-1}, \cdot) \quad \text{s.t.} \quad u_t^i = \gamma_t^i(m_t^i).$$

Let ψ denote the coordinator's strategy:

$$(\gamma_t^1, \ldots, \gamma_t^n) = \psi_t(z_{1:t-1}).$$

Virtual coordinator observes $z_{1:t-1}$ and prescribes $\boldsymbol{\gamma}_t := (\gamma_t^1, \ldots, \gamma_t^n) \in \mathcal{G}$.

An equivalent centralized POMDP [Nayyer et al., 2013]

A dynamic program is identified to characterize the optimal strategy based on the information state π.

$$V(\pi) = \min_{\boldsymbol{\gamma} \in \mathcal{G}} \mathbb{E}\left[\ell(x_t, \mathbf{u}_t) + V(\pi_{t+1}) | \pi_t = \pi, \boldsymbol{\gamma}_t = \boldsymbol{\gamma}\right].$$

Let $\mathcal{R}$ denote the reachable set of the information state π.

STEP 2: An Approximate POMDP RL Algorithm

Definition (Incrementally Expanding Representation)

Let $\{\mathcal{S}_N\}_{N=1}^{\infty}$ be a sequence of finite sets such that $\mathcal{S}_1 \subsetneq \mathcal{S}_2 \subsetneq \ldots \subsetneq \mathcal{S}_N \subsetneq \ldots$. Let $\mathcal{S} = \lim_{N\to\infty} \mathcal{S}_N$ be the countable union of above finite sets. The tuple $\langle \{\mathcal{S}_N\}_{N=1}^{\infty}, B, \tilde{f} \rangle$ is called an *Incrementally Expanding Representation*, if

Incremental Expansion: For any $\gamma \in \mathcal{G}, z \in \mathcal{Z}$, and $s \in \mathcal{S}_N$,

$$\boxed{\tilde{f}(s, \gamma, z) \in \mathcal{S}_{N+1}.}$$

Consistency: For any $(\gamma_{1:t-1}, z_{1:t-1})$, let π_t and s_t be the corresponding states at time t. Then,

$$\pi_t = B(s_t).$$

Pre-learning stage

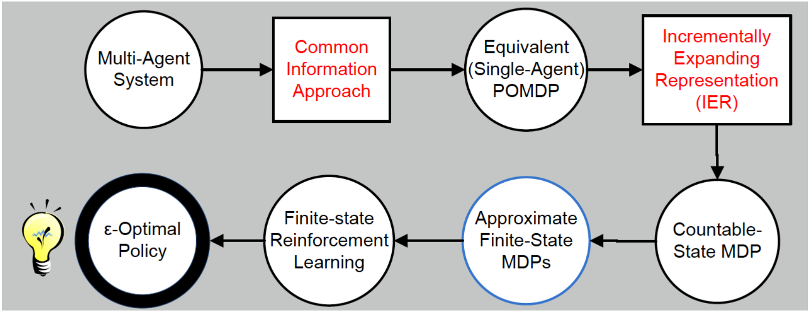
STEP 2: An Approximate POMDP RL Algorithm

Lemma

Every decentralized systems with PHS has at least one IER such that $\mathcal{S}$ and $\tilde{f}$ do not depend on unknowns .

- Construct countable-state MDP Δ with state space $\mathcal{S}$, action space $\mathcal{G}$, dynamics $\tilde{f}$, and cost $\tilde{\ell}(B(s_t), \gamma_t) := \mathbb{E}[\ell(x_t, u_t^1, \ldots, u_t^n)|\pi_t, \gamma_t]$.
- Construct an augmented type approximation sequence $\{\Delta_N\}_{N=1}^{\infty}$ of Δ, with state space $\mathcal{S}_N$, action space $\mathcal{G}$, dynamics $\tilde{f}$, and cost $\tilde{\ell}(B(s_t), \gamma_t)$.
- Apply a finite-state RL algorithm $\mathcal{T}$ (such as TD(λ) and Q-learning) to learn optimal strategy of Δ_N. We assume $\mathcal{T}$ converges to optimal strategy of Δ_N.

A Block Diagram



Proposed decentralized RL algorithm

(1) Given $\epsilon > 0$, choose N such that $\frac{2\beta^N}{1-\beta}(\ell_{max} - \ell_{min}) \leq \epsilon$. Then, construct Δ_N; particularly, state space $\mathcal{S}_N$ and dynamics $\tilde{f}$.

(2) At iteration k, ζ chooses prescriptions $\gamma_k = (\gamma_k^1, \ldots, \gamma_k^n)$. (Agents have access to a common random generator to explore consistently). Agent i takes action u_k^i based on prescription γ_k^i and local information m_k^i:

$$u_k^i = \gamma_k^i(m_k^i), \forall i.$$

(3) Based on taken actions, system incurs cost ℓ_k, evolves, and generates common observation z_k that is observable to every agent. Agents consistently compute next state as follows

$$s_{k+1} = \tilde{f}(s_k, \gamma_k, z_k) \in \mathcal{S}_N.$$

(4) $\mathcal{T}$ learns (updates) the coordinated strategy according to observed cost ℓ_k by performing prescriptions γ_k at state s_k and transition to state s_{k+1}.

(5) $k \leftarrow k + 1$, and go to step 2 until termination.

Proposed decentralized RL algorithm

(1) Given $\epsilon > 0$, choose N such that $\frac{2\beta^N}{1-\beta}(\ell_{max} - \ell_{min}) \leq \epsilon$. Then, construct Δ_N; particularly, state space $\mathcal{S}_N$ and dynamics $\tilde{f}$.

(2) At iteration k, ζ chooses prescriptions $\gamma_k = (\gamma_k^1, \ldots, \gamma_k^n)$. (Agents have access to a common random generator to explore consistently). Agent i

The structure of the learned strategy:

$$u_t^i = g_t^i(s_t, m_t^i), \quad i \in (1, \ldots, n),$$

where s_t is the internal state that changes every time.

$$s_{k+1} = \tilde{f}(s_k, \gamma_k, z_k) \in \mathcal{S}_N.$$

(4) $\mathcal{T}$ learns (updates) the coordinated strategy according to observed cost ℓ_k by performing prescriptions γ_k at state s_k and transition to state s_{k+1}.

(5) $k \leftarrow k + 1$, and go to step 2 until termination.

Learning stage: main theorem

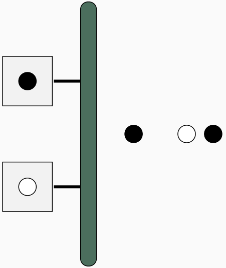
Theorem 6.3

Let J^* be the optimal performance of the original decentralized system and $\tilde{J}$ be the performance under the learned strategy. Then,

$$\tilde{J} - J^* \leq \epsilon_N,$$

where $\epsilon_N = \frac{2\beta^{\tau_N}}{1-\beta}(\ell_{max} - \ell_{min}) \leq \frac{2\beta^N}{1-\beta}(\ell_{max} - \ell_{min})$ and τ_N is a model dependent parameter that $\tau_N \geq N$.

Numerical example 2: multi Access Broadcast Channel (MABC)



- $x^i_t \in \{0, 1\}$ with independent arrival probability p^i, $i = 1, 2$.
- $I^i_t = (x^i_t, u^1_{1:t-1}, u^2_{1:t-1})$.
- $u^i_t \leq x^i_t \in \{0, 1\}$.
- In case of collision, packets remain in buffers.
- **Objective:** maximize the throughput.
  - State of other agent is unknown. (decentralized information)
  - Arrival probabilities are unknown. (incomplete model)

Future Work

Future work

- **Game theory**
- **Markov chain**
- **Reinforcement learning:** Specific teams such as mean-field teams.
- **Mean-field teams and consensus algorithms**
- **Various approximations in mean-field teams:** Information & model.
- **New model of mean-field teams**
- **Various applications:** Smart grids, communications, economics, robotics, social networks, etc.

Thank you.

Contributions

Main contributions: Mean Field Teams

- Introduce partially exchangeable agents and **mean-field teams**.
- Allow agents to be coupled in **dynamics** and cost under mild assumptions.
- Mean field sharing is **non-classical**. (difficult problems)
- We use novel approaches to find the **global** optimal solution.
- Solution approach works for **arbitrary** # of agents. (not necessarily large)
- Mean field can be computed and communicated easily or by local interactions using **consensus** algorithms.
- In large sub-populations, mean-field is **predictable**. Also, mean-field teams are **robust to node failure**.
- Different generalizations.

Main contributions: Mean Field Teams

- Introduce partially exchangeable agents and **mean-field teams**.

Salient features of mean-field teams:

1. Controlled Markov chain: solution complexity is **polynomial** (rather than exponential) in # of agents and **linear** (rather than exponential) in time.

2. Linear quadratic:
    - The optimal solution is **linear**.
    - The solution complexity is **independent of N** and it depends only K.
    - No need to share anything beyond mean field.
    - Each agent solves only two Riccati equations (**distributed computation**).

3. When population is infinite, mean-field is deterministic and computable.

Main contributions: Mean Field Teams

- Introduce partially exchangeable agents and **mean-field teams**.
- Allow agents to be coupled in **dynamics** and cost under mild assumptions.
- Mean field sharing is **non-classical**. (difficult problems)
- We use novel approaches to find the **global** optimal solution.
- Solution approach works for **arbitrary** # of agents. (not necessarily large)
- Mean field can be computed and communicated easily or by local interactions using **consensus** algorithms.
- In large sub-populations, mean-field is **predictable**. Also, mean-field teams are **robust to node failure**.
- Different generalizations.

Main contributions: Mean Field Teams

- Introduce partially exchangeable agents and **mean-field teams**.
- Allow agents to be coupled in **dynamics** and cost under mild assump-

- **Arbitrarily coupled cost**
- **Infinite horizon**
- **Noisy observation**
- **Major-minor**
- **Randomized strategies**

- **Weighted mean field**
- **Infinite horizon**
- **Partial mean-field sharing**
- **Major-minor**
- **Tracking problem**

teams are **robust to node failure**.

- Different generalizations.

Main contributions: Reinforcement Learning with PHS

- There is **no existing RL** in team that guarantees optimality .
- Introduce a novel decentralized RL for partial history sharing that **guarantees $\epsilon-$optimal** solution.
- Use **common information approach** and our **proposed approach** to design the learning space.
- Introduce the notion of **Incrementally Expanding Representation**.
- The proposed approach is also **novel in centralized POMDP**.
- Develop **decentralized Q-learning** for two-user MABC.

Main contributions: Reinforcement Learning with PHS

- There is **no existing RL** in team that guarantees optimality .
- Introduce a novel decentralized RL for partial history sharing that **guar-**

Three features of designed learning space $\mathcal{S}_N$:

- It is implementable by every agent based on common knowledge.
- It takes into account of the model and cost (not a prefixed space).
- It adapts to the exiting powerful finite state-action RL algorithms.

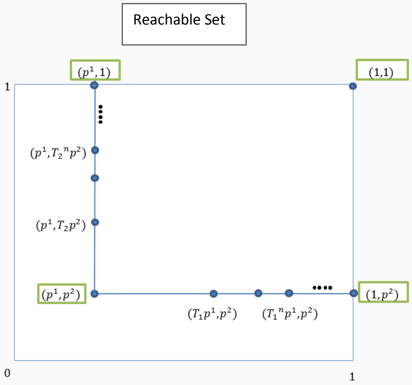
- The proposed approach is also **novel in centralized POMDP**.
- Develop **decentralized Q-learning** for two-user MABC.

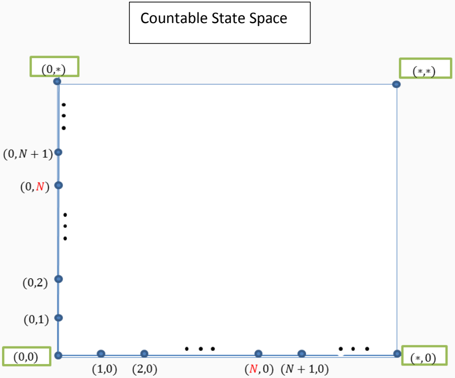
Main contributions: Reinforcement Learning with PHS

- There is **no existing RL** in team that guarantees optimality .
- Introduce a novel decentralized RL for partial history sharing that **guarantees $\epsilon-$optimal** solution.
- Use **common information approach** and our **proposed approach** to design the learning space.
- Introduce the notion of **Incrementally Expanding Representation**.
- The proposed approach is also **novel in centralized POMDP**.
- Develop **decentralized Q-learning** for two-user MABC.

Reinforcement Learning: Multi-Access Broadcast Channel

Numerical example 2: MABC





Both users transmit   Only user 1 transmits   Only user 2 transmits

Mean-field team: temperature control

Numerical example 3: temperature control